**Homework 11**

**P10.35** Determine **I1**, **I2**,  **V1**, and **V2** in Figure P10.35.

**Solution:** From KVL on the input side in Figure P10.35,

(3 + *j*4)**I1** + **V1** = **VSRC**; from KVL on the output

side, (12 – *j*5)**IO** = **V2**; for the transformer, **V2** = 5**V1**; 400**IO** + 100**I2**= 0, or, **IO** = **I1** + **I2**. These equations give: (12 – *j*5)**I1** – 25**V1** = 0. Solving with the first equation:

**I1** = = 1.851 - *j*0.584 A; **V1** = 10(cos30° + *j*sin30°) = 0.772 – *j*0.651 V. **V2** = 5**V1** = 3.86 – *j*3.25 V, **I2** = *I*1 = -1.481 + *j*0.467 A. **IO = I1 + I2** = 0.37 – *j*0.117 ≡ 0.388∠-17.5° A.

**P10.38** Determine *iS*(*t*) in Figure P10.38, assuming that *vSRC*(*t*) = 20cos1000*t* V.

**Solution:** 1/*jωC* = -*j*/(103×25×10-6) = -*j*40 Ω. The circuit in the frequency domain is as shown in Figure P10.38A, with the voltages and currents assigned as indicated. From the mmf equation, 100**IS** + 50**I2** – 250(**IS** – **I2**) = 0, which gives **IS** = 2**I2**; From KVL in the upper mesh, **VSRC** = 3**V1** + 6**IS**; From KVL around the two lower windings: **V1** – 16(**IS** – **I2**) + 5**V1**

+ *j*40(**IS** – **I2**) = 0; substituting for for **I2** gives: 3**V1** = (4 – *j*10)**IS**; substituting for 3**V1**: **VSRC** = (4 – *j*10)**IS** + 6**IS** = 10(1 –*j*)**IS**. It follows that **IS** = 20∠0°/10(1 – *j*) = ∠45° A, so that *iS*(*t*) = cos1000(*t* + 45°) A.

**P10.41** Derive TEC looking into terminal ‘ab’ in Figure P10.41.

**Solution:** On open circuit in Figure P10.41A, 50 = 30**I1** – **VTh**/2, where **VTh**/4 = -2**I1**(-*j*10). Substituting for **I1** and simplifying gives **VTh** = (-64 + *j*48) V.

On short circuit in Figure P10.42B, the voltages across all windings are zero, which makes the current in the upper secondary winding zero. The current in the primary winding is 2**ISC**; KVL gives: 50 = -2**ISC**×30, so that **ISC** = -5/6 A. It follows that *ZTh* =  Ω.

**P10.43** Determine **IX** in Figure P10.43.

**Solution:** Because of the short circuit, the voltages across all windings are zero. The currents are therefore as indicated in Figure P10.43A. From

the mmf equation, 400×2.5 – 100×5 – 200(**IX** – 5) = 0, or 10 – 5 – 2**IX** + 10 = 0, which gives **IX** = 7.5 A.

**P10.47** Determine *k* in Figure P10.47 so that no current flows in *ZX*.

**Solution:** When no current flows in *ZX* in Figure P10.47A, **Va** = **Vb**, and the current in the output coil of the linear transformer is zero. It follows that **I** = **VI**/*jω*120, and **V*a*** = (*jωM*/*jω*120)**VI** = (*M*/120)**VI**. Moreover, **V*b*** = (400/2000)**VI**.

Setting **V*a*** = **V*b***, *M* = 120/5 = 24 mH. It follows that  0.4.

**P10.48** Determine *X* in Figure P10.48 so no current flows in the 5 Ω resistor, assuming *a* = 2.

**Solution:** When the current in the 5 Ω resistor is zero, the resistance can be removed, resulting in the output, RHS coil of the linear transformer having no current and a voltage **VI** across it, as shown in Figure P10.48A. The input current of the coil on the LHS of the linear transformer is **VI**/*jωM* and voltage across this coil is (**VI**/*jωM*)*jωL*1 = **VI**, since *M* = *L*1. This is also the secondary voltage of the ideal transformer. It follows that the primary voltage of this transformer is **VI**/a, and the primary current is a**VI**/*jωM.* Hence,*****j*10(0.5 – 0.25) = *j*2.5, which makes *X* = 2.5 Ω.

**P10.51** Derive TEC looking into terminals ‘ab’ in Figure P10.51, where **VTH** = **Vab**.

**Solution:** Replacing the two coupled coils by the T-equivalent circuit, the circuit becomes as shown in Figure P10.51A. The mmf equation is: 100**I** – 200**I** = 0, so that **I** = 0. It follows that **Va** = *j*30**ISRC**, and **V1** = -*j*50**ISRC**. Hence, **Vb** = -2**V1** = *j*100**ISRC**. This gives **VTh** = *j*30**ISRC** – *j*100**ISRC** = -*j*70**ISRC** = -*j*7 V = 7∠-90° V.

To determine *ZTh*, a test current source **IT** is applied, with the source **ISRC** replaced by an open circuit. The circuit becomes as shown in Figure P10.51B. The mmf equation is:100**I** – 200(**I** – **IT**) = 0, so that **I** = 2**IT**. From KVL around the outer loop, **V1** = *j*30(**I** – **IT**) + *j*20**I** = *j*70**IT**. From KVL around the upper mesh, **Vab** = (-*j*10)**IT** + *j*20**I** + **V1**. Substituting for **I** and **V1** in terms of **IT**, **Vab** = *j*30**IT** + *j*70**IT** = *j*100**IT**. Hence, *ZTh* = *j*100 Ω.

**P10.52** Determine *a* in Figure P10.52 so that *Yin* = 0, assuming *ω* = 1 Mrad/s

**Solution:** The two coupled coils in Figure P10.52A have *Leq* = 6 + 4 – 2×3 = 4 μH, and an impedance of *jωLeq* = *j*4 Ω *;* 1/*jωC* = 1/(*j*106×0.25×10-6) = -*j*4 Ω.

 When a test source **VT** is applied, the test current **IT** should be zero. The voltage across *jωLeq* is (**VT** – *a***VT**), so that **I** = (1 – *a*)**VT/**(*j*4). From KCL at the upper node, *a***I** = **I** + **VT/**(-*j*4), or **I** = **VT/**[(*j*4)(1 – a)]. Equating the two expressions of **I**: , or, , *a* = 1 ± 1, or, *a* = 2.